Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/42**

Paper 4 Mechanics **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\pm\frac{1}{2}\times 15\times 2^2\,[=\pm 30]$ $\pm\frac{1}{2}\times 15\times 4^2\,[=\pm 120]$	**B1**	For KE at top or bottom. Need not be evaluated. $\frac{1}{2}\times 15\times(4-2)^2$ is B0.
	$\pm 15g\times 1.6\,[=\pm 240]$	**B1**	For PE change. Need not be evaluated.
	$240+30=120+W$	**M1**	Attempt at work energy equation; 4 relevant terms; dimensionally correct; allow sign errors. $\frac{1}{2}\times 15\times(4-2)^2$ is M0. If $W = F$ times a numerical distance seen, then M0.
	Work done = 150 J	**A1**	

Question	Answer	Marks	Guidance
1	**Alternative method for Q1**		
	$4^2 = 2^2 + 2a \times \dfrac{1.6}{\sin\theta}$	***M1**	Attempt to use $v^2 = u^2 + 2as$ with $s = \dfrac{1.6}{\sin\theta}$ or $\dfrac{1.6}{\cos\theta}$ but not $s = 1.6\sin\theta$ or $1.6\cos\theta$ or 1.6. If θ is given a value, then M0. Must be using speeds 2 and 4 here.
	$15g\sin\theta - R = 15a$	**DM1**	3 terms; allow sign errors; allow sin/cos mix but weight must be resolved; dimensionally correct.
	$R = 93.75\sin\theta$	**A1**	$R = 93.75\cos\theta$ Must be consistent with their s.
	Work done $\left[= 93.75\sin\theta \times \dfrac{1.6}{\sin\theta}\right] = 150$ J	**A1**	
		4	

Question	Answer	Marks	Guidance
2	Attempt to resolve in at least one direction to form an equation.	***M1**	Correct number of terms; allow sign errors; allow sin/cos mix; allow with different T 's.
	$T\sin 30 + T\sin 40 - 2 = 0$	**A1**	If different T's then allow M1A1A0 max.
	$T\cos 30 - T\cos 40 - mg = 0$	**A1**	Allow with their T.
	Attempt to solve for T or m	**DM1**	From equation(s) with correct number of relevant terms.
	Tension $T = 1.75,\ m = 0.0175$	**A1**	$T = 1.7501\ldots\ \ m = 0.017497\ldots$ awrt 1.75 for T www, and awrt 0.0175 for m www.
		5	

Question	Answer	Marks	Guidance
3	Work done by 120 N force = $120 \times 5\cos 20 \left[= 563.81557\ldots.\right]$	**B1**	
	$\left(\text{PE change} =\right) 10g \times 5\sin 30 \left[= 250\right]$	**B1**	For PE change.
	Attempt at work energy equation	**M1**	4 relevant terms; dimensionally correct; allow sign errors; allow sin/cos mix in relevant resolved terms.
	$120 \times 5\cos 20 - 10g \times 5\sin 30 - 200 = \frac{1}{2} \times 10 \times v^2$ $\left[563.815\ldots - 250 - 200 = 5v^2\right]$	**A1**	
	Speed = 4.77 m s^{-1}	**A1**	awrt 4.77.

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3**		
	Resistive force = $\frac{200}{5}\left[=40\right]$	***B1**	oe e.g. $5\times RF=200$.
	$120\cos 20-RF-10g\sin 30=10a$	***M1**	4 relevant terms; dimensionally correct; allow sign errors; allow sin/cos mix; allow with their resistive force or just RF.
	$a=2.276\ldots$	**A1**	Allow arwt 2.3 to 2sf from correct work.
	$v^2=0+2\times(2.276\ldots)\times 5$	**DM1**	Use of $v^2=u^2+2as$ using $u=0$, $s=5$ and their positive a which has come from a resistive force using work done.
	Speed = $4.77\,\text{m}\,\text{s}^{-1}$	**A1**	awrt 4.77.
		5	

Question	Answer	Marks	Guidance
4(a)	$R = 0.2g$	**B1**	
	$1.2 = \mu \times 0.2g$	**M1**	Resolve horizontally and using $F = \mu R$ to get an equation in μ; 2 relevant terms.
	$\mu = 0.6$	**A1**	oe
		3	
4(b)	$1.2 - 0.3 \times 0.2g = 0.2a$	***M1**	Resolve horizontally using Newton's Second Law; 3 relevant terms; allow sign errors; $R = 0.2g$ only.
	$a = 3$	**A1**	$0.6 = 0.2a$ only seen, allow with BOD, but if 0.6 as friction being used as resultant force, this is M0A0.
	$s_3 = 0 + \frac{1}{2} \times 3 \times 3^2 \left[= 13.5 \right] s_2 = 0 + \frac{1}{2} \times 3 \times 2^2 \left[= 6 \right]$	**DM1**	For use of $s = ut + \frac{1}{2}at^2$ (or a complete method) to find a distance at least once with $u = 0$ and their positive a and $t = 2$ or $t = 3$.
	Distance = $13.5 - 6 = 7.5$ m	**A1**	www
		4	

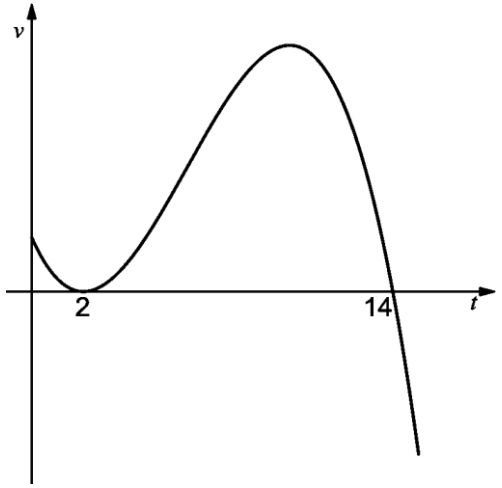
Question	Answer	Marks	Guidance
5(a)	$v^2 = 25^2 + 2(-g) \times 20$ OR $\frac{1}{2} \times 0.5 \times v^2 = \frac{1}{2} \times 0.5 \times 25^2 - 0.5 \times g \times 20$	**M1**	Use of $v^2 = u^2 + 2as$ with $u = 25$, $s = 20$ and $a = \pm g$. OR using change in KE = ± change in PE.
	Speed $= 15\ \text{m s}^{-1}$	**A1**	
		2	
5(b)	Taking up as positive direction: $0.5 \times 15 + 0.3 \times (-32.5) = 0.5v + 0$ or Taking down as positive direction: $0.5 \times (-15) + 0.3 \times 32.5 = 0.5v + 0$	**M1**	For use of conservation of momentum, 3 non-zero terms, allow sign errors, using their speed $15\ \text{m s}^{-1}$. Must show how ± 2.25 is obtained.
	[Taking up as positive direction: velocity of $A = -4.5\ \text{m s}^{-1}$] [Taking down as positive direction: velocity of $A = 4.5\ \text{m s}^{-1}$] Speed = 4.5 m s^{-1} direction downwards	**A1**	Any error seen in calculating v is A0. Must explicitly say 4.5 m s^{-1} and downwards.
		2	

Question	Answer	Marks	Guidance
5(c)	Downwards to be positive, for A $20 = 4.5t_A + \frac{1}{2}gt_A^2$ and solve for t_A Upwards to be positive, for A $-20 = -4.5t_A - \frac{1}{2}gt_A^2$ and solve for t_A	**M1**	Using constant acceleration formula(e) to get a correct equation in t_A and solve for t_A. If using quadratic formula, must be the correct formula. If factorising, when brackets expanded, 2 terms correct.
	For B $20 = 0 + \frac{1}{2}gt_B^2$ $[t = 2]$ and solve for t_B	**M1**	Using constant acceleration formula(e) to get a correct equation in t_B and solve for t_B.
	$t_A = 1.6$ or $t_B = 2$	**A1**	
	Difference = 0.4 s only	**A1**	
		4	

Question	Answer	Marks	Guidance
6(a)	Engine: $125000 - 120000g \times 0.02 - 22000 - T = 120000a$ $125000 - 120000g\sin(1.145\ldots..) - 22000 - T = 120000a$ $[125000 - 24000 - 22000 - T = 120000a \Rightarrow 79000 - T = 120000a]$ Coach: $T - 60000g \times 0.02 - 13000 = 60000a$ $T - 60000g\sin(1.145\ldots..) - 13000 = 60000a$ $[T - 12000 - 13000 = 60000a \Rightarrow T - 25000 = 60000a]$ System: $125000 - 120000g \times 0.02 - 60000g \times 0.02 - 22000 - 13000 = (120000 + 60000)a$ $125000 - 120000g\sin(1.145\ldots..) - 60000g\sin(1.145\ldots..) - 22000 - 13000 = (120000 + 60000)a$ $[125000 - 24000 - 12000 - 22000 - 13000 = (120000 + 60000)a \quad \Rightarrow 54000 = 180000a]$	***M1**	Attempt at Newton's second law at least once; correct number of relevant terms; allow sign errors; allow sin/cos mix; allow g missing; a value for α or $\sin\alpha$ must be substituted. Allow with $\alpha = 1.1$ or better $[\alpha = 1.145991998]$.
		A1	Any equations correct.
		A1	Two equations correct. If using separate equations for engine and coach and different T's, then allow M1A1A0 max.
	Solve for T or a	**DM1**	Using equations with the correct number of relevant terms. If no working seen, must be solutions to their equation(s) to be awarded M1.
	Acceleration = $0.3\,\text{m s}^{-2}$ and Tension = 43 000 N	**A1**	Allow 0.299 from use of $\alpha = 1.15$. Awrt 43000 to 3sf from correct work.
		5	

Question	Answer	Marks	Guidance
6(b)	Driving force, $DF = \frac{4500000}{30}\left[=150000\right]$	**B1**	Use of $F = \frac{P}{v}$, oe e.g. $DF \times 30 = 4500000$.
	Attempt to resolve parallel to the track once if using system equation, twice if using equations for engine and coach separately	**M1**	Correct number of relevant terms; allow sign errors; allow sin/cos mix; allow g missing. Must be correct number of equations depending on method.
	System: $150000 - 120000g\sin\beta - 60000g\sin\beta - 22000 - 13000 = 0$ or for Engine: $150000 - 120000g\sin\beta - 22000 - T' = 0$ and Coach: $T' - 60000g\sin\beta - 13000 = 0$	**A1**	Allow *DF* or their *DF*. Must be using same T'.
	Solve to get $\beta = 3.7°$	**A1**	3.663058552 awrt 3.7° www.
		4	

Question	Answer	Marks	Guidance
7(a)	Attempt to differentiate v	***M1**	Decrease power by 1 and a change in coefficient in at least one term (which must be the same term); allow unsimplified; allow p or q for t. $a = \frac{v}{t}$ is M0.
	$\left(a = \frac{\mathrm{d}v}{\mathrm{d}t} =\right) 3\times -0.1t^{3-1} + 2\times 1.8t^{2-1} - 6t^{1-1} = -0.3t^2 + 3.6t - 6$	**A1**	May be unsimplified. Allow p or q for t.
	Setting $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 0$ and attempt to solve a 3 term quadratic for t. $\left[a = \frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow 3t^2 - 36t + 60 = 0 \Rightarrow t^2 - 12t + 20 = 0\right]$	**DM1**	Allow p or q for t. Must get 2 values or numerical expressions for t from their three term quadratic. If using quadratic formula, must be the correct formula. If factorising, when brackets expanded, 2 terms correct.
	$p = 2,\ q = 10$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(b)	Velocities are $0\,\text{m s}^{-1}$ and $25.6\,\text{m s}^{-1}$	**B1**	SOI
	Curve with single minimum turning point followed by single maximum turning point	***B1**	Ignore placement of graph on axes Not a cusp for the minimum point or maximum point.
		**DB1**	All correct in 1st and 4th quadrant. Must go from convex to concave. Need to label 2 and 14 on the t-axis where the curve meets the t-axis. Do not need to show exact velocities at $t = 0$ or $t = 10$ or $t = 15$. Ignore graph outside $0 \leqslant t \leqslant 15$.
		3	

Question	Answer	Marks	Guidance
7(c)	Attempt to integrate v	***M1**	Increase power by 1 and a change in coefficient in at least one term (which must be the same term); $s = vt$ is M0.
	$(s =) -\frac{0.1}{4}t^{3+1} + \frac{1.8}{3}t^{2+1} - \frac{6}{2}t^{1+1} + 5.6t\,(+c)$ $= -0.025t^4 + 0.6t^3 - 3t^2 + 5.6t \ [+c]$	**A1**	May be unsimplified.
	Attempt distance from $t = 0$ to $t = 14$ [= 176.4]	**DM1**	Correct use of limits 0 and 14 for their s, i.e. $F(14) - F(0)$ May see limits 0 to 2 and 2 to 14 used but must be $\big(F(14) - F(2)\big) + \big(F(2) - F(0)\big)$.
	Attempt distance from $t = 14$ to $t = 15$ [= (-) 8.025]	**DM1**	Correct use of limits 14 and 15 for their s, i.e. $\pm\big(F(15) - F(14)\big)$. For reference $F(2) = \frac{18}{5} = 3.6$, $F(14) = \frac{882}{5} = 176.4$ and $F(15) = \frac{1347}{8} = 168.375$.
	Total distance $= 176.4 + 8.025 = 184.425$ m $= \frac{7377}{40}$	**A1**	www Condone 184 or better.

Question	Answer	Marks	Guidance
7(c)	**SC for those who show no integration. Max 3 marks.**		
	$\int_0^{14}\left(-0.1t^3+1.8t^2-6t+5.6\right)\mathrm{d}t=176.4$	**B1**	
	$\int_{14}^{15}\left(-0.1t^3+1.8t^2-6t+5.6\right)\mathrm{d}t=-8.025$ OR $\left\|\int_{14}^{15}\left(-0.1t^3+1.8t^2-6t+5.6\right)\mathrm{d}t\right\|=8.025$	**B1**	
	Total distance $=176.4+8.025=184.425$ m $=\frac{7377}{40}$	**B1**	Condone 184 or better.
	SC for those who show no integration and don't consider the 2 areas. Max 1 mark.		
	$\int_0^{15}\left\|-0.1t^3+1.8t^2-6t+5.6\right\|\mathrm{d}t=184.425$ m $=\frac{7377}{40}$	**B1**	Condone 184 or better.
		5	